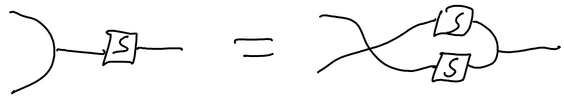
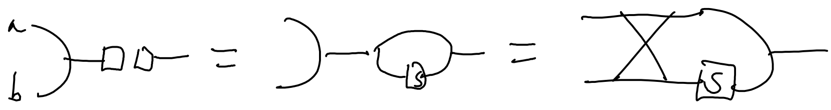
Peter Lee on December 7, 2009

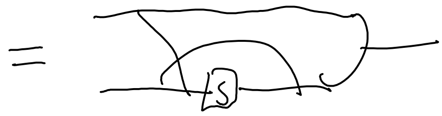
December-07-09
9:41 AM

Start: An improved proof that in a HA, the antipode is an anti-homomorphism; i.e., that



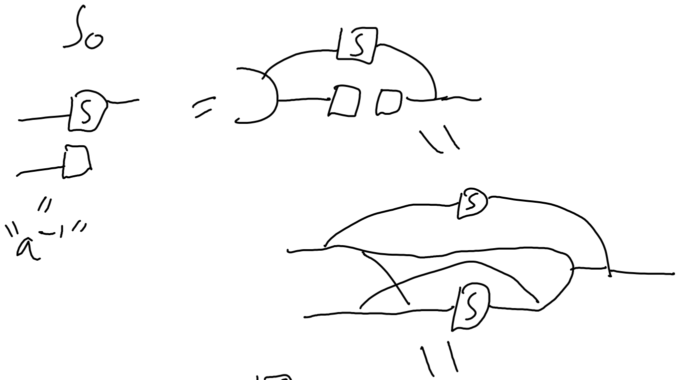
Pf



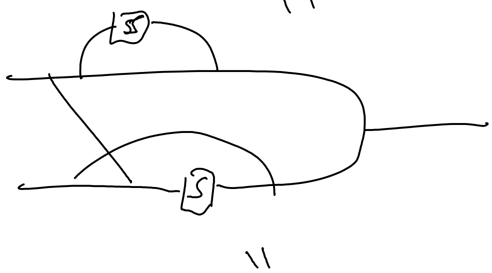


(This is, with $c=(ab)^{-1}$
$a(bc)=e$)

So

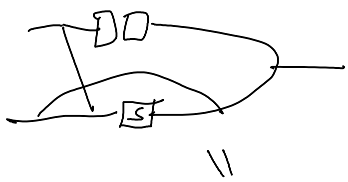


multiply by a^{-1} on left.

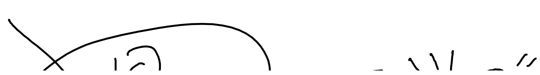


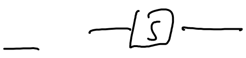
re-associate

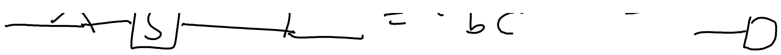
use antipode



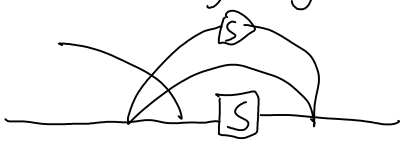
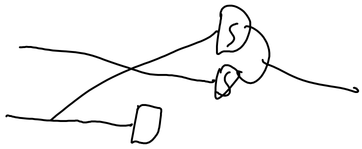
$=$ "$e\cdot(bc)$"







Now multiply by b^{-1} on the left

 = 

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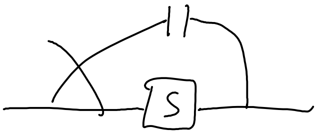 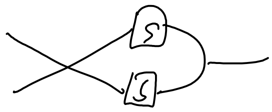

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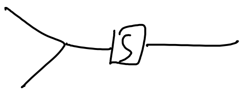
 QED

<u>Def</u> A HA A is "top free" if ... 1. $A = V[[h]]$

missing details 2. $A \otimes A \to A \hat{\otimes} A$

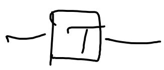
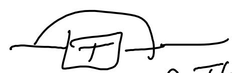
<u>Def</u> A is a deformation of A_0 if $A/hA = A_0$ as HA.

<u>Def</u> A is a QUEA if $A/hA \cong U(\mathfrak{g})$ as H.A.

<u>Claim</u> Any BA H w/ $H/hH = A$ as B.A. has a unique compatible antipode making H a HA w/ $H/hH \cong A$ as HA.

given a HA A.

<u>subclaim</u> $L : \mathrm{End}\, A \to \mathrm{End}\, A$ by (A here is a HA)

 $\to$ 

$T(a)$ $\qquad$ $a\,T(a)$

L_A is invertible & L_H is filtered, and the deg 0 piece of L_H is L_A (the L of A)

$\Rightarrow$ So L_H is invertible.

set $S_H = L^{-1}(\eta \circ \epsilon)$ ... QED